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# THE EFFECTS OF ANTENNA HEIGHT ON OTHER ANTENNA PROPERTIES

*A computer study*

While examining the properties of some interesting antennas based on the extended double Zepp (EDZ), I had occasion to review a 12-meter phased array consisting of two EDZ elements cut to the standard formula (0.64 wavelength each side of center feed), spaced  $1/8$  wavelength (4'11"), and fed 180 degrees out of phase. At 35 feet, a reference dipole using copper wire showed a calculated gain of 6.9 dBi, while a single wire EDZ showed a gain of 9.6 dBi. The phased array had a gain of 12.6 dBi, which was 3.0 dB better than the simple EDZ and 5.7 dB better than the simpler dipole. I was surprised at the relatively large gain of the array over the single wire antennas. It wasn't consistent with results I had calculated for variations on the EDZ theme. Then it hit me: 35 feet was  $7/8$  of a wavelength on the 12-meter band. In examining the characteristics of antennas at low heights typical of those used by amateurs with limited funds and space, I had learned that the results of calculations performed at  $7/8$  wavelength weren't always consistent with those achieved above and below that height.

To confirm my suspicion, I performed a

series of calculations via ELNEC for a collection of 12-meter wire antennas at heights of 20 to 70 feet (in 1-foot increments) above medium ground (average earth), using copper wire losses.\* My present stock of models, logged into a Quattro spreadsheet file for convenience, includes dipoles, 2 and 3-element Yagis, 2-element 180-degree phased arrays, 135-degree phase-fed beams, and delta and quad loop beams. Data gathered includes main-lobe take-off angle, gain, feed-point impedance, and front-to-back ratio, wherever relevant. Gain figures use the lowest main lobe at its maximum. Except for the lowest heights investigated (20 to 25 feet), the differences in take-off angles varied too little to note in the body of this study, but see **Appendix I** for a few notes on the subject. In any event, this study makes no claim as to the DX performance of any antenna. In fact, many of the models are far from optimized, having been chosen to test hypotheses related to their properties at various heights.

Taking the time to study the patterns of

\*ELNEC is available from Roy Lewallen, W7TL, P.O. Box 6638, Beaverton, Oregon 97007.

**ANTENNA HEIGHTS BETWEEN 1/2 AND 1 1/2 WAVELENGTH,  
20 THROUGH 10 METERS**

Frequency in MHz	Antenna Height in Feet per 1/2 Wavelength Increase										
	1/2	3/4	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
14.2	34.63	43.29	51.95	60.61	69.27	77.92	86.58	95.24	103.90	112.56	121.21
18.11	27.16	33.94	40.73	47.52	54.31	61.10	67.89	74.68	81.47	88.26	95.04
21.2	23.20	29.00	34.80	40.60	46.39	52.19	57.99	63.79	69.59	75.39	81.19
24.95	19.71	24.64	29.57	34.50	39.43	44.35	49.28	54.20	59.13	64.06	68.96
28.5	17.26	21.57	25.88	30.20	34.51	38.83	43.14	47.45	51.77	56.08	60.39

Table 1. 1/2-wavelength increments of antenna height for a selected frequency within each ham band from 20 through 10 meters.

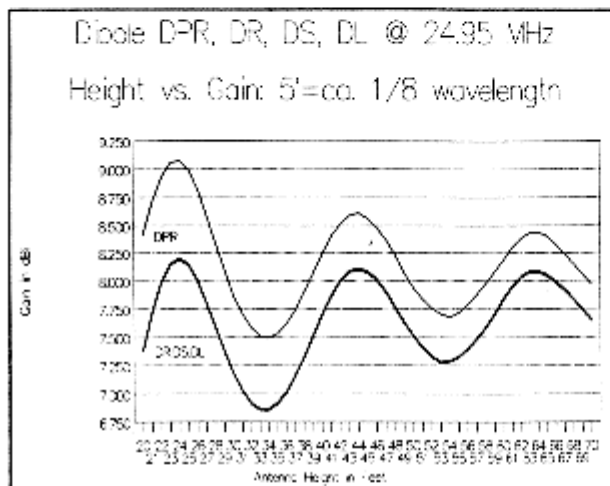


Figure 1. Gain variations in dipoles DPR, DR, DS, and DL from 20 to 70 foot heights.

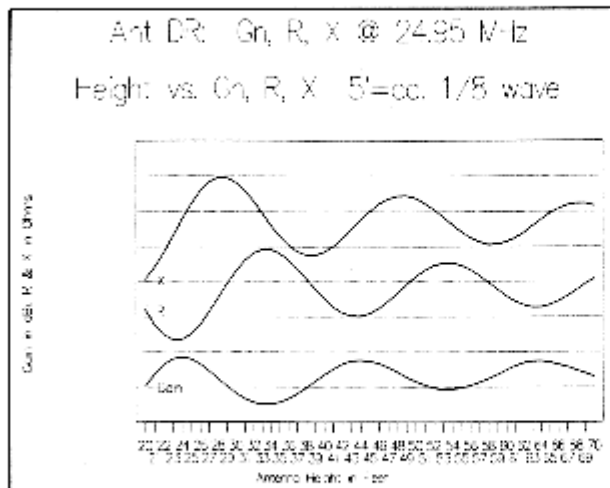


Figure 2. Gain versus resistance and reactance for a 1/2-wavelength dipole from 20 to 70 foot heights.

antenna properties as the height of the antenna is varied has proven very instructive to me. Antenna specifications, whether presented by hams or commercial manufacturers, usually appear as a single set. Sometimes writers use free space numbers, sometimes real earth numbers. Some specify gain in dBi, others use dBd, still others use a real dipole at the same height as their standard. None, however, specify antenna performance over a range of heights, but perhaps they should. In any event, if we understand how the performance of various types of antennas varies with height, we can have more realistic expectations when we build or buy an antenna and install it at home.

Twelve meters is an interesting band to use for calculating antenna performance at the heights of typical amateur installations. In the United States, we tend to work in increments of 5 feet, which is close to 1/8 wavelength at 12 meters. Table 1 lists the heights that correspond to 1/8-wavelength increments from 1/2 to 1-3/4 wavelengths for a selected point in each of the ham bands from 20 through 10 meters. The results derived for the antenna models at 12 meters can be translated to other bands with adjustments for height. The effects of yard clutter, uneven terrain, and other variables limit the precision with which models can be realized in practice. So, too, do construction practices. Hence, gain and other figures are useful only for *generalized* comparisons. Their absolute values are relatively unimportant. Indeed, most of the information in this study appears in graphic form, as the shape of the curves may be more educational than tables of numbers from which the graphs derive. For reference, each antenna model type is accompanied by one or more free space azimuth pattern. Where relevant, some elevation patterns also appear. In the end, the line graph gives you an indication of antenna performance.

Applying the figures from this study to other bands requires conversion by refer-

ence to wavelengths and fractions of wavelengths above ground. Extrapolating the 12-meter results to other bands is necessarily limited by at least two factors. First, practical building limitations restrict extrapolations to 20 through 10 meters. On 6 and above, the lowest antennas are typically higher than 1-3/4 wavelengths up. Similarly, on 80, the highest antennas are below the half-wavelength point. Second, the average or medium soil model from which these figures result becomes nonlinear at the lower HF frequencies. To this second factor we may add the problem of changing depths to which antenna currents penetrate in the lower HF region, which troubles any assumption of coherent soils underlying an antenna system. **Appendix 2** provides some additional details on these limitations of extrapolation.

Any study like this, undertaken in spare time, is subject to specific data point errors. Modeling each antenna for between four and seven data points per step and 51 steps per model opens the door to transcription errors. Only gain numbers were allowed the three decimal places given by ELNEC in order to smooth gain curves. Other data were rounded on the fly to one decimal place—except for the takeoff angle and certain very high reactances, which were recorded as integers. A second opening for transcription error occurs in manually entering the data into a spreadsheet for analysis (a time con-

suming and fatiguing task).<sup>\*</sup> Simple antennas, like the dipoles, took about 1 minute per step to run and record; simplified quad and delta loops took about 3.5 minutes per step. Fully tapered-element loop antennas took close to 9 minutes per step, even with a coprocessor on a 20-MHz computer; therefore, they were only spot checked for coincidence with the substitute models. Nonetheless, what I learned in the process made the fatigue worthwhile, and I apologize in advance for any data point errors.

## Dipoles

As long as I can remember, amateur literature on 1/2-wavelength dipoles has recorded the fact that the feedpoint resistance and reactance change as one moves the antenna upward. Less prominent (indeed, invisible) in most literature is the fact that dipole gain, as a function of a comparison to an isotropic source, also changes with height. In fact, gain minima and maxima may be greater than 1.3 dB apart. Unlike NBS standards for length and weight, the dipole is a highly variable standard.

<sup>\*</sup>For those who would like to examine the data, a copy of the spreadsheet file is available, if you can read a compressed Quattro file on your own spreadsheet. To receive the file, send a perforated IBM disk in a self-addressed envelope with sufficient return postage. If you send a disk capable of more than 600 KB, I shall supply the uncompressed file as well, if you require it. Regrettably, I can not format the file for other spreadsheet systems or to non-IBM-compatible computers, nor can I take the responsibility for the readability of the file. I can only copy the file and hope for the best.

DIPOLFS: (all antennas computed at 30 segments per wire)								
Antenna Description	Designator	Material	Element	Length	Free Space Gain dBi	Characteristics R	Characteristics X	Source
Dipole, resonant perfect ground	DPR	#18 Cu	Only	19.2 feet	2.066	73.5	+1.9	Formula
Dipole, resonant	DR	#18 Cu	Only	19.2 feet	2.066	73.5	+1.9	Formula
Dipole, short	DS	#18 Cu	Only	19.0 feet	2.057	73.1	+13.6	Formula
Dipole, long	DL	#18 Cu	Only	19.4 feet	2.075	75.9	+17.5	Formula
Dipole, thick	DT	1/4-inch Al	Only	19.4 feet	2.116	76.3	+21.0	Formula
Dipole, very thick	DVT	1/4-inch Al	Only	19.4 feet	2.156	79.8	+27.5	Formula
Extended Double Zepp	EDZ	#18 Cu	Only	25.25 feet	4.905	124.9	+678	Formula

Table 2. Dipoles modeled in this study.

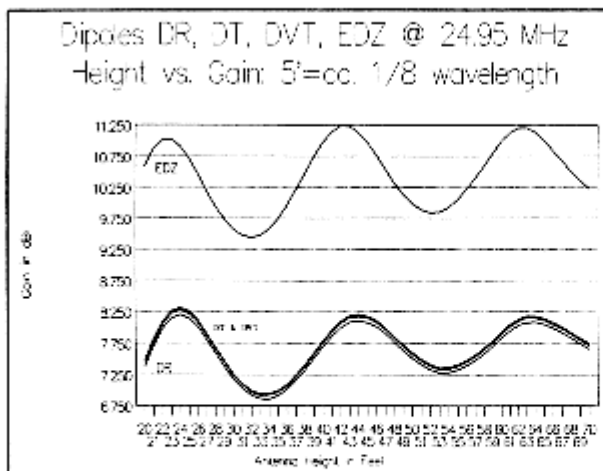


Figure 3. Gain variations in dipoles DR, DT, DVT, and EDZ from 20 to 70 foot heights.

Table 2 lists the dipole models evaluated via ELNEC. The reason for the large number of models is simple: they answer some interesting newcomer questions about variations in dipole performance. First, does the quality of ground make a difference in the position of maxima and minima? Second, does the length of a 1/2-wavelength dipole make a difference? Antenna models DPR, DR, DS, and DL answer these two questions unequivocally. No! Figure 1 graphs the gain of four no. 18 copper dipoles at 24.95 MHz over medium ground. The upper curve traces the gain change of DPR, the dipole over perfect ground. We may note in passing that gathering data on any antenna over perfect ground is more difficult than over real ground. The reason is that higher angle lobes may show higher gain figures than the lowest lobe when above perfect ground. The current state of MININEC programs re-

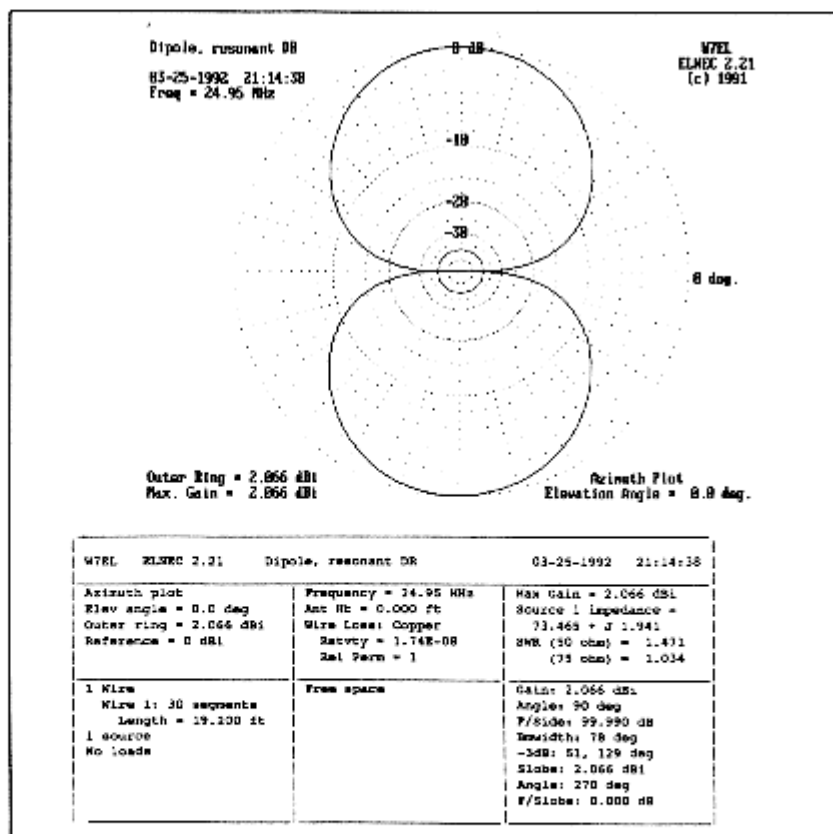


Figure 4. Free space pattern for a 1/2-wavelength dipole.

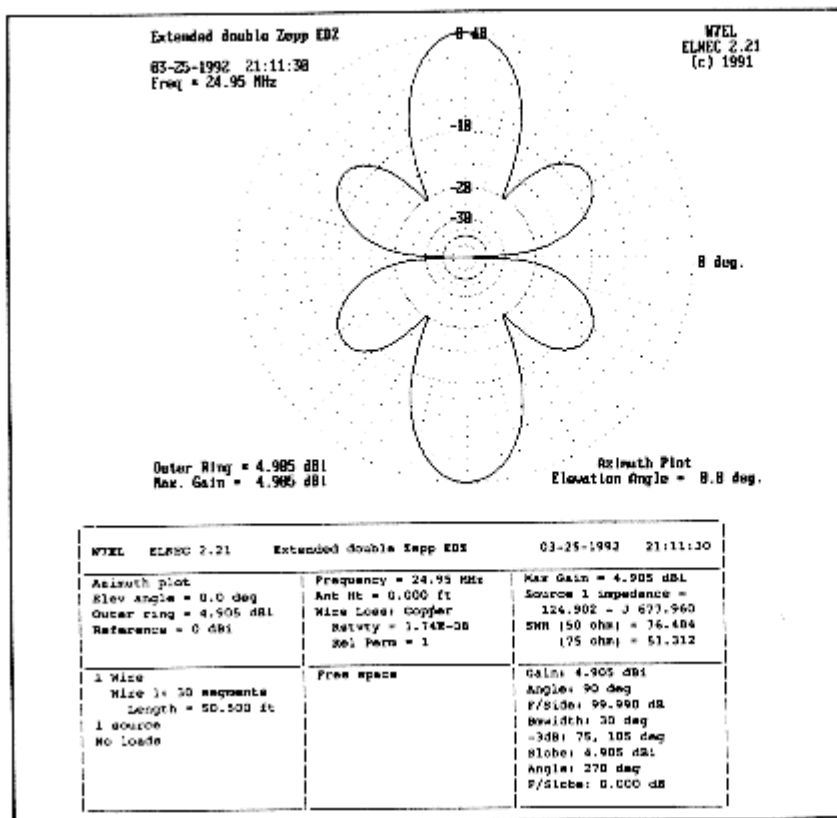


Figure 5. Free space pattern for an extended double Zepp dipole.

quires that one go hunting for the gain of the lowest lobe.

The lower curve of Figure 1 is actually three traces so close together as to be inseparable. Antenna DR is roughly resonant in the sense of having its reactance alternate between capacitive and inductive as the antenna goes up. DS presents a varying capacitive reactance at all heights, while DL presents an inductive reactance at all heights. Regardless of whether the antenna is slightly short or slightly long, the gain maxima and minima remain in the same places. Small but significant amounts of reactance don't displace these points any more than does the nature of the ground. In general, for 1/2-wavelength dipoles, the gain maxima coincide with the minima of the resistive component of the feedpoint impedance, as shown in Figure 2.

A related question concerns the effect of element thickness upon maxima and minima. Antennas DT and DVT used 1/8-inch

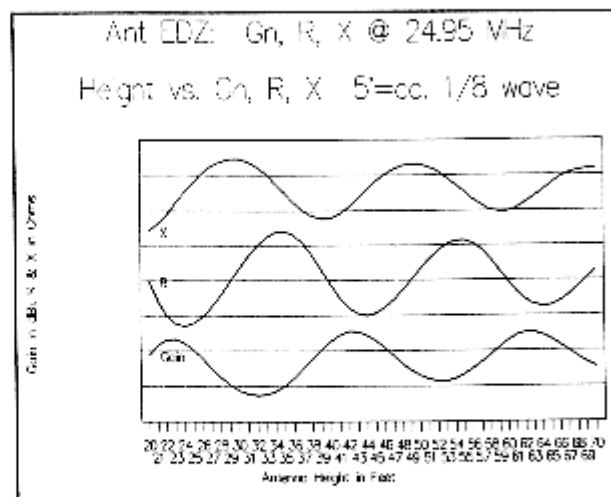


Figure 6. Gain versus resistance and reactance for an extended double Zepp dipole from 20 to 70 foot heights.

**180-DEGREE PHASE-FED ARRAYS: (all antennas computed at 10 segments per half wavelength)**

Antenna Description	Designator	Material	Element	Length	Space to Previous Element	Free Space Characteristics			Source
						Gain dBi	R	X	
Double WRJK 180 degree phase fed	DRJKP	#18 Cu	DE Refl.	19.7 feet 19.7 feet	4.9 feet	6.847	1455	-6786	Formula
Double EDZ 180 degree phase fed	DEDZP	#18 Cu	DE Refl.	25.25 feet 25.25 feet	4.9 feet	7.695	19.3	-654	HB, 92, p 33-11
Double Quad Loop 180 degrees phase fed	DQLp	#18 Cu	DE Refl.	40.32 feet 40.32 feet	5.0 feet	5.830	19.7	+8.5	Formula 1005/f
Double Quad Loop 180 degrees phase fed	DQLp-A	#18 Cu	DE Refl.	(41.60 feet) (41.60 feet)	5.0 feet	5.880	20.2	+6.5	Sub. model for DQLP 1038/f

Notes: HB = 1992 ARRL Handbook.

Table 3. 180-degree phase-fed arrays modeled in this study.

and 1-inch diameter aluminum elements, respectively. Figure 3 shows the results of modeling these thick and very thick dipoles. Although gain increases enough to be barely visible, nothing else changes in the variation of gain as a function of antenna height.

For resonant or near-resonant 1/2-wavelength dipoles, the actual positions of the gain maxima and minima are slightly short of true 1/8-wavelength points. Maxima occur near, but before the 5/8, 1-1/8, and 1-5/8 wavelength points, while minima occur just before the 7/8 and 1-3/8 wavelength points. The difference is roughly 1 foot at 12 meters, or about 0.025 wavelength. The next question is whether this holds true for all dipole antennas.

Figure 4 shows the free space pattern of a 1/2-wavelength dipole, with its well-known pinch-waisted pattern. Figure 5 shows a much longer dipole, the extended double Zepp (EDZ), also modeled on no. 18 copper wire. Each leg of this dipole is about 0.64 wavelength long. Because the antenna is nonresonant, it exhibits a large reactance at the feedpoint. At a total length of 5/4 wavelengths, the reactance is capacitive. This effective antenna shows a displacement of maxima and minima between 2 to 3 feet lower—more than 0.06 wavelength lower—than the 1/2-wavelength antenna, as seen in Figure 3. As Figure 6 shows, the maxima and minima are not directly related to either

the resistance or reactance at the feedpoint, but to intermediate points.

Another more subtle difference between the 1/2-wavelength dipole and EDZ appears in Figure 3. All 1/2-wavelength dipoles display their highest gain at about 5/8 wavelength and their lowest gains at 7/8 wavelength. The graph approximates the voltage curve for a dampened oscillator. Above 3 wavelengths of antenna height, the difference between maxima and minima drops to about 0.2 dB or less. The EDZ follows a slightly different gain pattern, reaching its maximum level at 1-1/8 wavelength. The increase of gain with height between the 5/8 and 1-1/8 wavelength points is common for many other antennas that lack the oscillation of values shown by the dipoles. The EDZ shows characteristics of being a mixed-breed antenna.

There are some lessons to be learned from even the limited analysis performed here. The lessons apply to almost any antenna design one might disseminate to others. In the real world of antennas, there are perhaps no standard antennas, not even the 1/2-wavelength wire dipole. There are only *ceteris paribus* references; that is, references if all other things are equal. Hence, even the wire dipole is not a blank standard for horizontal antennas. Rather, the dipole serves as a reference at the same frequency, at the same height, over the same type of

earth, in the same orientation, and made of the same material. As a baseline for comparisons, it is not a number, but a graph against which other antennas can be plotted.

### 180-degree phase-fed arrays

If dipole antenna configuration has little or no effect upon the variations in main lobe gain until we reach the very high reactance of the EDZ, even though feedpoint resistance and reactance display similar but displaced variations, then another question arises. What creates the variations in gain? The question receives a partial answer from the group of antennas listed in Table 3. All the antennas have two elements, fed 180 degrees out of phase with each other. The double W8JK is a classic and has been built with spacing from 1/8 to 1/4 wavelength. The double EDZ is of more recent vintage and appears in *The ARRL Handbook*. The double quad loop is a conceptual invention

designed to add one more antenna to the lot; its performance doesn't justify construction by anyone. In fact, because loops require so much computer time to run when accurately modeled (using the element tapering feature of ELNEC), I ran a substitute. Its parenthetical model-only dimensions allowed me to track the same performance within reasonable limits while using only six segments per wire.

Figures 7 and 8 show the free space azimuth patterns for the D8JKp and DEDZp antennas, respectively. Analysis of the dimensions of the double W8JK would show its elements to be slightly long under any conditions, even though they are traditionally considered to be 1 wavelength long. A precise 1-wavelength dimension for any given amateur frequency would yield an antenna without reactance at that frequency. However, even a small frequency excursion would cause a reactance jump. Similarly, small changes of length, such as those

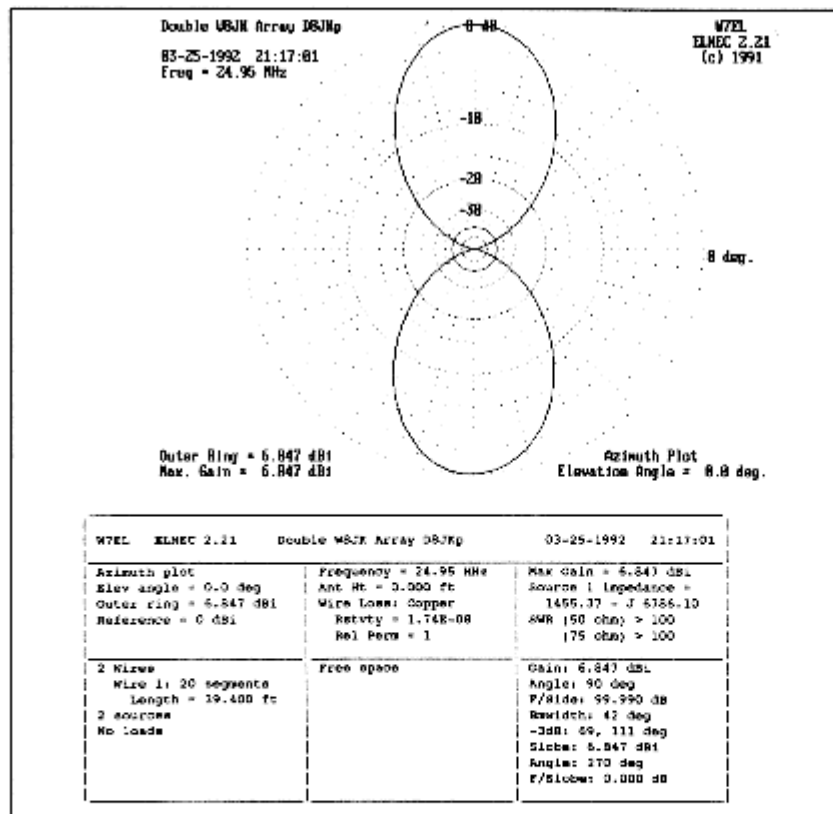


Figure 7. Free space pattern for a double W8JK array.

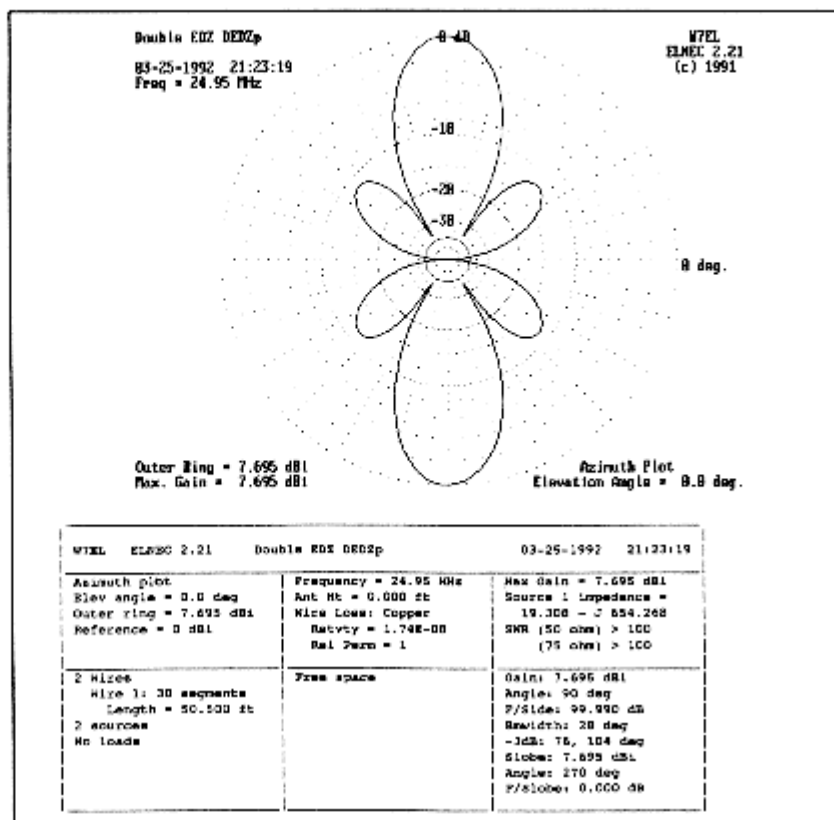


Figure 8. Free space pattern for a double extended Zepp array.

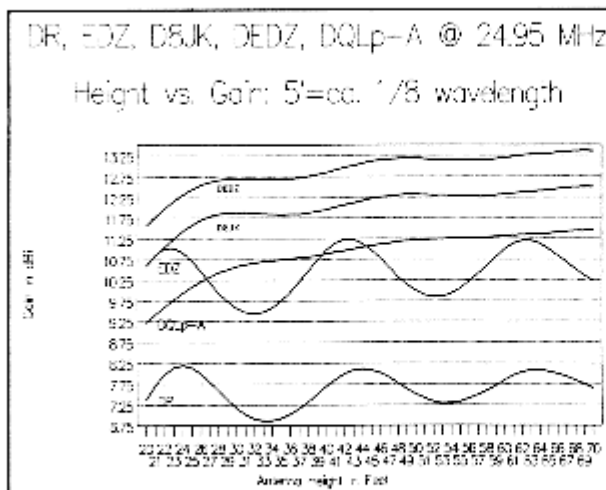


Figure 9. Gain variations in antennas DR, EDZ, D8JK, DEDZ, and DQLp-A from 20 to 70 foot heights.

created by element sag due to gravity, create the same effect. Varying the model element length between 37.6 and 37.8 feet produced a resistive feedpoint impedance component around 30,000 ohms, while the reactive component went from 10,700 ohms inductive to 4,400 ohms capacitive—a change of more than 15,000 ohms in about 2-1/2 inches. The longer 39.4-foot elements produce a large, but stable, capacitive reactance without pattern distortion. In contrast, the double EDZ array has more gain, but a more complicated pattern of radiation.

Figure 9 shows the results of modeling these arrays with their 180-degree feed systems. Resonant 1/2-wavelength dipole and EDZ patterns are shown for the contrast. With all three phase-fed arrays, the gain patterns show little peaking and may be considered "well-behaved." These same patterns also show up in the impedance figures. Above three quarters of a wavelength (30 feet at 12 meters), both the resistive and



reactive components of the double quad loop vary less than 1 ohm each. The peaks and valleys in the patterns of the other two arrays vary only slightly, but do show an inverted co-variance with the resistive component. That is, feedpoint resistance peaks as gain dips. Figure 10 shows the phenomenon for the double W8JK. The double EDZ isn't worth graphing in this regard, since its reactance varies back and forth by 1 ohm throughout the height range investigated.

There seems to be a pretty good reason for the lack of gain and impedance variation among the antennas of this group. Figure 11 shows the elevation pattern of the double EDZ antenna at the 40-foot level. Compare the high angle radiation immediately above the antenna to Figure 12, the elevation pattern of the resonant dipole at the same height. Without high angle radiation to reflect off the ground and back to the antenna, variations in antenna currents and phase angles, with consequential altera-

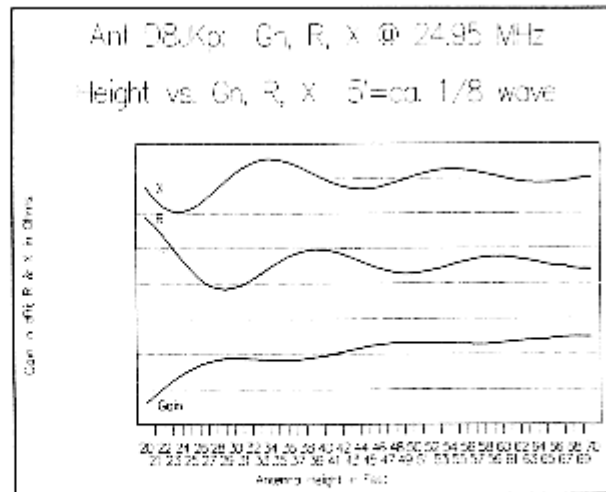


Figure 10. Gain versus resistance and reactance for a double W8JK array from 20 to 70 foot heights.

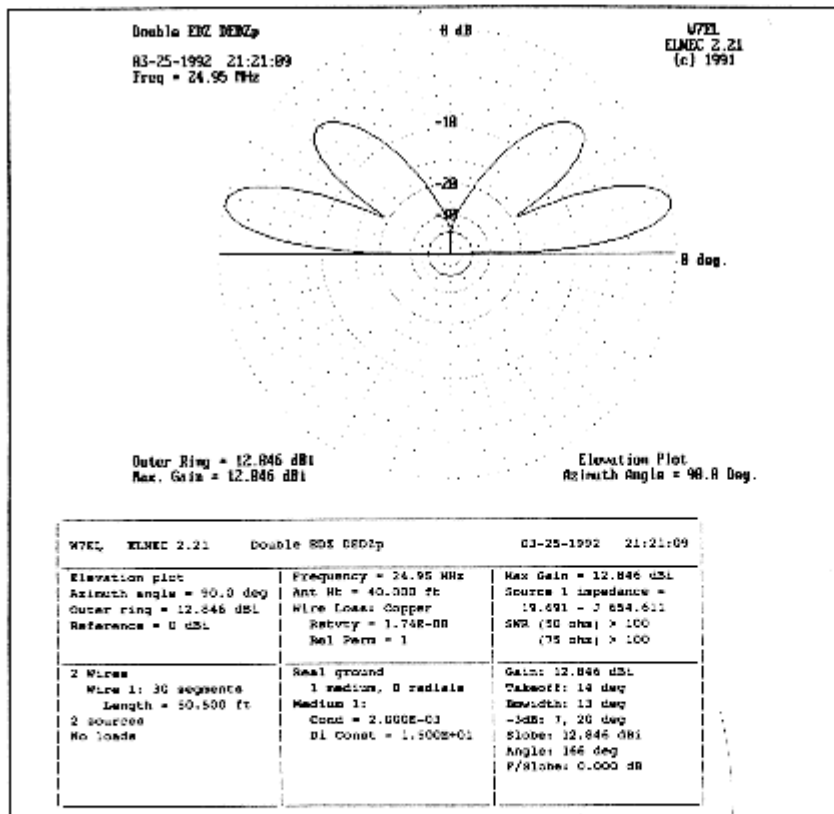


Figure 11. Elevations pattern of the double EDZ array at a height of 40 feet.



COMPARISON OF ANTENNA GAIN EXTREMES AT 2 SELECTED HEIGHTS									
Height Feet	Wavelength	Gain Dipole	Gain EDZ	Gain Double WJK	Gain Double EDZ	WJK Over Dipole	DEDZ Over Dipole	WJK Over EDZ	DEDZ Over EDZ
35	5/8	6.91	9.73	11.83	12.68	4.92	5.77	2.10	2.95
45	1-1/8	8.98	10.97	12.22	13.11	4.14	5.03	1.25	2.14
Difference in Gain Advantage Due to Height Change from 35 to 45 feet:						0.78 dB	0.74 dB	0.85 dB	0.81 dB

Table 4. Comparison of antenna gain extremes at 2 selected heights.

wavelength) and a wide-spaced model (a bit under 1/4 wavelength). The close-spaced model had been designed for a gamma match by Bill Orr, W6SA1, and showed considerable capacitive reactance. The wide-spaced model proved to be a close match for 50-ohm coax, with a consequent reduction in both gain and front-to-back ratio.

The results of setting the beams through 1-foot steps appear in Figure 13. The gain curves for both Yagis are "well-behaved," with only small ups and downs. Like the phased arrays, the curves show a relatively steady increase in gain with height (contrary to the collection of dipoles). It's interesting to note that the small peaks for both antennas occur in the same place as those for dipoles: about 0.025 wavelength prior to the 5/8, 1-1/8, and 1-5/8 wavelength points (25, 45, and 65 feet at 12 meters). Gain maxima coincide closely with the lowest values for the resistive component of the feedpoint impedance. However, the minima occur up to 3 feet earlier. Nonetheless, the presence of a parasitic element appears to

protect the gain from much of the variation induced in dipoles.

Front-to-back ratio, however, is another matter. As Figure 13 demonstrates, both Yagis show great fluctuations in front-to-back ratios as height increases, although the maxima and minima decrease with height. Roughly, the peaks and valleys occur at the 1/4-wavelength marks. Y2R-2, the 50-ohm model with near resonance, more closely hits those marks, while the heavily reactive model, Y2R-1, leads by a consistent foot (0.025 wavelength).

The director models of the 2-element Yagi were designed by reference to formulas taken from two different handbooks without regard to whether they were good antennas to build. Their overall gain and front-to-back ratio figures compare favorably with the reflector models, but matters such as bandwidth were not checked. Both models are fairly close-spaced, with one excelling in gain, the other in front-to-back ratio. More importantly, one was significantly capacitively reactive, the other in-

2-ELEMENT YAGI BEAMS										
Antenna Description	Designator	Material	Element	Length	Space to Previous Element	Free Space Characteristics				Source
						Gain dBd	F-B dB	R	X	
2-element Yagi DE + Refl.	Y2R-1	1-inch Al	DE	17.8 feet		6.716	10.1	23.0	-27.7	CQ, 12-90, p. 83, scaled
				19.6 feet	4.8 feet					
2-element Yagi DE + Refl.	Y2R-2	1-inch Al	DE Refl.	18.2 feet		6.442	8.7	50.2	+0.3	AB, 16 Ed, p. 11-2 II
				19.4 feet	8.7 feet					
2-element Yagi DE + Dir.	Y2D-1	1-inch Al	Dir. DE	18.04 feet		6.640	10.2	27.7	-9.7	RHb, 21 Ed, p. 29.4
				19.08 feet	4.81 feet					
2-element Yagi DE + Dir.	Y2D-2	1-inch Al	Dir. DE	18.50 feet		7.208	8.8	18.4	+24.0	AB, 16 Ed, p. 11-7
				19.88 feet	3.94 feet					

Notes: CQ - CQ Magazine, December, 1990, p. 83; AB - ARRL Antenna Book, 16th Ed.; RHb - Radio Handbook, 21st Ed.

Table 5. 2-element Yagi beams modeled in this study.

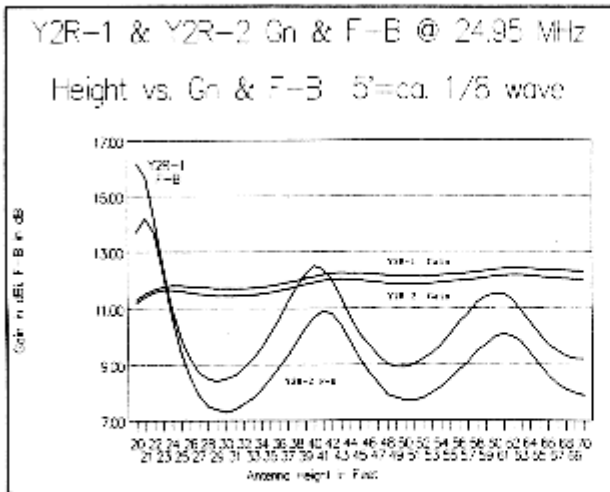


Figure 13. Gain and front-to-back ratio versus height for two 2-element Yagis with reflectors.

ductively reactive. For reference and comparison, Figures 14 and 15 show free space patterns of Y2R-1 and Y2D-1.

Figure 16 displays the results of modeling the director Yagis. Like their reflective cousins, these antennas display well-behaved gain curves, with maxima and minima closely placed at the 1/8-wavelength positions. However, the gain maxima and minima of these directive Yagis coincide directly with the peaks and valleys of the resistive component of the feedpoint impedance, in direct opposition to both dipoles and 2-element Yagis with reflectors. The higher the antenna, the more closely feedpoint resistance coincides with gain peaks.

Again, like their reflective cousins, the directive Yagis show front-to-back ratios that vary widely over the range of antenna height. Y2D-2, the significantly inductive model, reaches its peaks 1 to 2 feet ahead of the gain peak. The capacitively reactive model tends to be late, reaching its front-to-

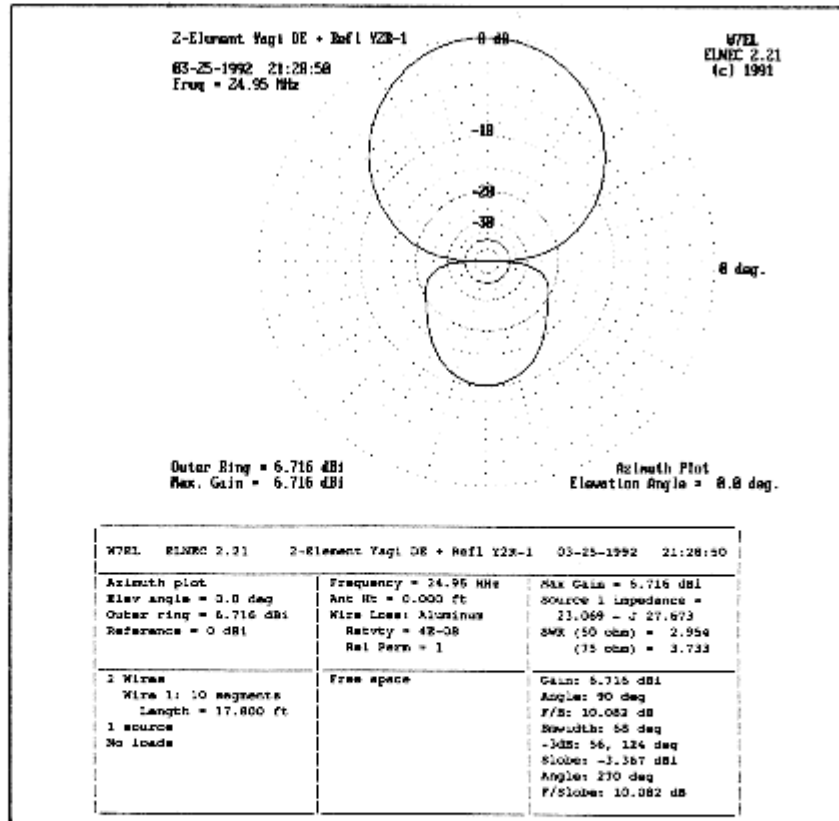


Figure 14. Free space pattern for Y2R-1, a 2-element Yagi with reflector.

back ratio peaks after the gain peak. In this model, where the reactive component is approximately equal to the resistive component, the front-to-back ratio peaks coincide even more closely with the resistive peaks than the gain peaks do. In the case of Y2D-1, where the feedpoint resistance is about 3 times the reactance, the gain, front-to-back ratio, and resistance peaks tend to cluster together.

The most significant difference between the reflective and the directive Yagis is the position of the front-to-back ratio peaks. Roughly speaking, they are out of phase with each other. Where the directive Yagi peaks its front-to-back ratio, the reflective Yagi hits a valley. We may ignore gain, which changes little with height, but like the arrays, climbs slowly. A directive 2-element Yagi will tend to show a better front-to-back ratio than its reflective cousin in the following ranges: 25 to 30 feet, 45 to 50 feet, and 65 to 70 feet ( $5/8$  to  $3/4$ ,  $1-1/8$  to

$1-1/4$ , and  $1-5/8$  to  $1-3/4$  wavelength high). The reflective 2-element Yagi shines at under 25 feet, 35 to 42 feet, and 55 to 62 feet ( $1/2$  to  $5/8$ ,  $7/8$  to 1, and  $1-3/8$  to  $1-1/2$  wavelengths high). These performance notes, of course, are relative to the general performance capabilities of 2-element Yagis.

### 3-element Yagis

The 3-element Yagi holds the potential for much superior performance with respect to both gain and front-to-back ratio. In this day of computer-optimized Yagis, I had some difficulty coming up with a variety of designs to test. Table 6 shows the three models used. The first, Y3-1, is scaled from an *ARRL Antenna Book* design. The second, Y3-2, uses formulas from an older handbook and represents a wide-spaced model. The third, Y3-3, derives from formulas in the ARRL book, but strives for equal close-spaced elements. Figure 17 pre-

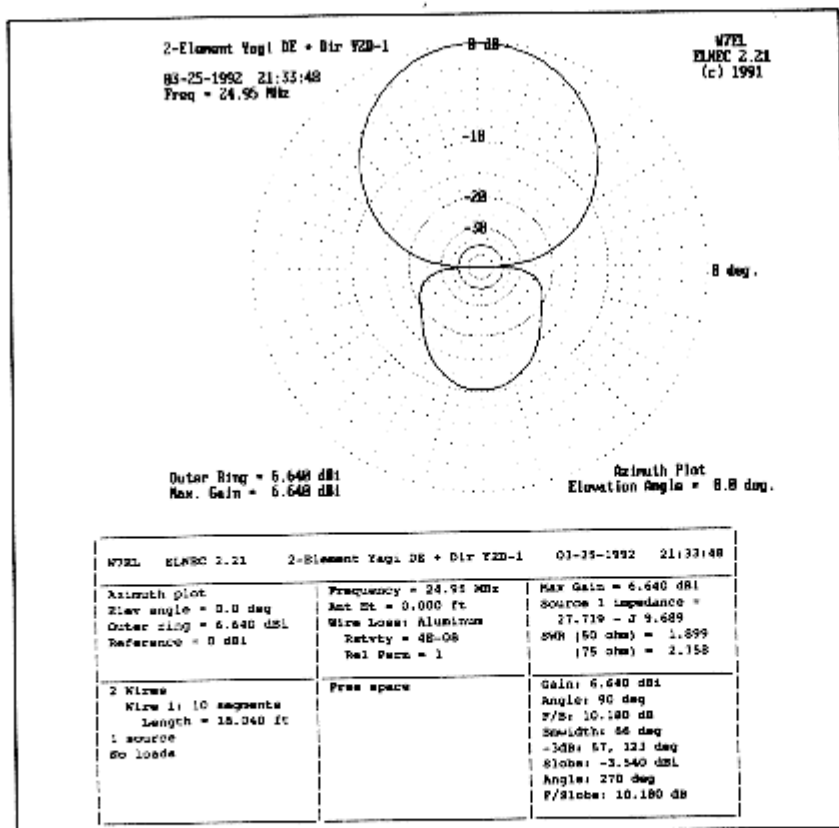


Figure 15. Free space pattern for Y2D-1, a 2-element Yagi with director.

sents a free space azimuth pattern of Y3-3 for reference. The three models together yield both higher and lower resistive components to the source impedance and both inductive and capacitive reactance, distributed among the various models.

The 3-element Yagi is a fairly complex antenna in terms of element interaction. Many builders have despaired of having gain, front-to-back ratio, and bandwidth merge. Other properties of the antenna also diverge as one changes dimensions. Interestingly, the model antennas tend to split according to whether their elements are close-spaced or wide-spaced.

Like the 2-element Yagis and the phased arrays, the gain of the 3-element Yagis climbs rapidly between the 1/2-wavelength height and the 3/4-wavelength point, as shown in Figure 18. For models Y3-1 and Y3-3, the close-spaced beams, the gain continues to rise, changing only in the rate of increase. The wide-spaced beam, Y3-2, shows an overall increase in gain, but passes through peaks and valleys in the process. In fact, its rapid rise phase is delayed by 1/8 wavelength compared to the other 3-element Yagis. It would appear that wide element spacing does not permit the parasitic elements as effectively to isolate the gain from the effects of reflected high angle radiation. This condition is also confirmed in the variations of feedpoint resistance and reactance, both of which vary by up to 20 percent. In contrast, the close-spaced Yagis exhibit a total resistance and reactance range of around 1.5 ohms, which reduces to a range of 1 ohm or less above the 3/4-wavelength height.

Turning to Figure 19, we discover an ob-

verse condition in the front-to-back ratios. The wide-spaced beam, Y3-2, shows the least variation in front-to-back ratio values, although they are the lowest of the group. The close-spaced Yagis exhibit significant maxima and minima, with the optimized model, Y3-1, showing sharp peaks just above the progressive half-wave heights. Interestingly, both the close-spaced beams have front-to-back maxima that coincide closely with the feedpoint resistance maxima, while Y3-2 shows a reasonable coincidence between inductive reactance maxima and front-to-back ratio peaks.

The complex interactions among the elements of these Yagis permit no unqualified generalizations. Element spacing around 1/8 wavelength produces beams with certain consistent characteristics, but those properties change as the element spacing approaches a quarter wavelength. Whether even computer optimization can produce a 3-element Yagi that performs consistently with respect to gain and front-to-back ratio at all reasonable heights may be doubtful. The lesson, if any, is this: a beam optimized for one height requires reoptimization before installation at another.

### 135-degree phase-fed antennas

An interesting class of antennas consists of two elements, the rear of which is fed 135 degrees out of phase with the front. Standard element spacing is 1/8 wavelength. Traditionally made of twinlead elements with a twisted 1/8-wavelength twinlead phasing line from the front element,

#### 3-ELEMENT YAGI BEAMS:

Antenna Description	Designator	Material	Element	Length	Space to Previous Element	Free Space Characteristics				Source
						Gain dBi	FB dBi	R	X	
3-element Yagi handbook design	Y3-1	1-inch Al	Dir.	18.6 feet		7.957	16.5	8.9	-7.9	AB, 16 Ed., p. 11-11
			DE	19.0 feet	4.0 feet					
			Ref.	19.8 feet	6.0 feet					
3-element Yagi 0.25 wave-length	Y3-2	1-inch Al	Dir.	18.04 feet		8.649	7.3	38.1	+48.5	Rhb, 21 Ed., p. 29.6
			DE	18.96 feet	9.86 feet					
			Ref.	20.08 feet	9.86 feet					
3-element Yagi 0.15 wave length spacing	Y3-3	1-inch Al	Dir.	18.44 feet		8.771	12.6	9.8	+10.4	AB, 16 Ed., p. 11-11
			DE	19.05 feet	5.91 feet					
			Ref.	19.88 feet	5.91 feet					

Notes: AB = ARRL Antenna Book, 16th Ed.; Rhb = Radio Handbook, 21st Ed.

Table 6. 3-element Yagi beams modeled in this study.

these are the notorious ZL-Specials. They come in two varieties. ELNEC originator Roy Lewallen, W7EL, created his Field Day Special by using two elements of equal length. Older ZL-Special designs tended to make the rear element longer to optimize gain and front-to-back ratio. A typical ZL-Special free space pattern appears in Figure 20. Both models appear in Table 7: the W7EL model (scaled) as FDSP, the older model as ZLSP.

In modeling the ZL-Specials, I followed the lead of ELNEC's originator and made each element from a single fat wire, 0.145 inches in diameter. This simulates the thickness of twinlead without the difficulties inherent in directly modeling closely-spaced parallel wires. However, the resultant feed-point resistance and reactance values will not be those associated with twinlead models. The patterns of rise and fall, if any, will parallel twinlead values.

In general, the Field Day and ZL-Special

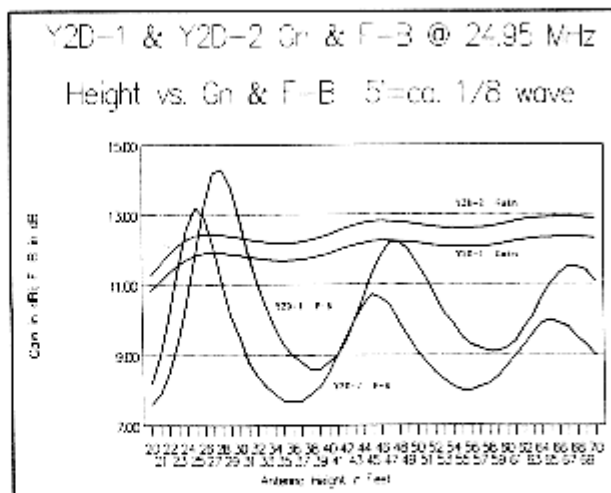


Figure 16. Gain and front-to-back ratio versus height for two 2-element Yagis with directors.

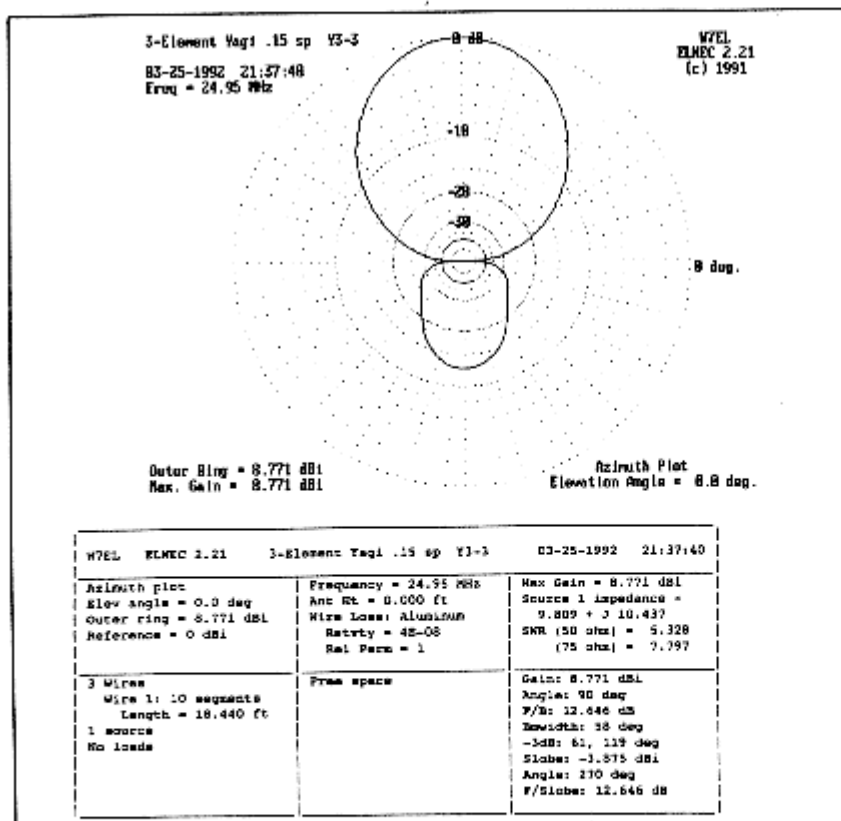


Figure 17. Free space pattern for Y3-3, a 3-element Yagi with 0.15 wavelength element spacing.

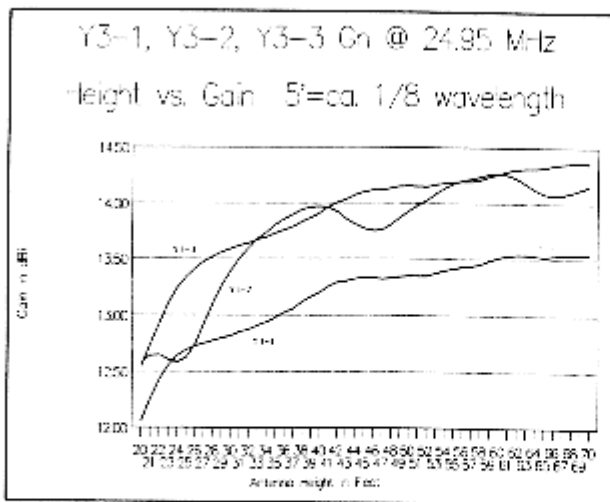


Figure 18. Gain variations in Yagis Y3-1, Y3-2, and Y3-3 from 20 to 70 foot heights.

variations have little to distinguish them. The traditional ZLSP shows a marginally higher gain and a seemingly significant increase in front-to-back ratio at any height, but that is an artifact of comparing an idealized model to a scaled actual antenna.\*

\*W3FL uses a program capable of evaluating Field Day and ZL Special designs using equal or unequal element lengths, so long as the elements are folded dipoles. The program yields element current calculations that are very accurate, as verified by actual element current measurements. His work is testimony to the fact that, while we may never eliminate the cut-and-try aspect of antenna construction, dedicated antenna specialists with a knack for computer programming can get us much closer to precision performance than at any time in radio history.

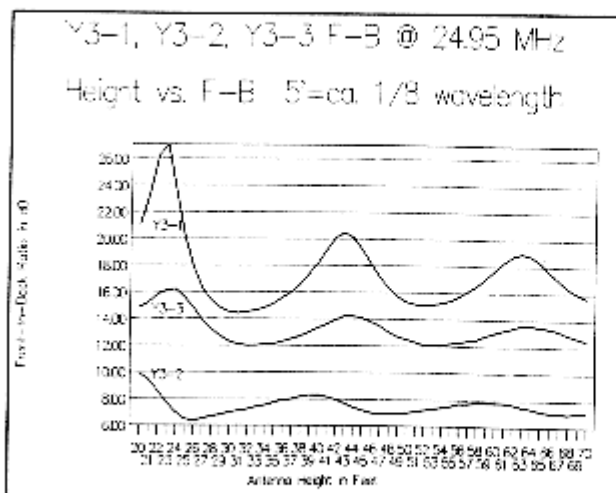


Figure 19. Front-to-back ratio variations in Yagis Y3-1, Y3-2, and Y3-3 from 20 to 70 foot heights.

Moreover, front-to-back ratios higher than about 20 dB may be of little use unless an offending station is aligned directly to the rear of the antenna. ZL-Specials show two rear side lobes, down about 20 dB from the main forward lobe. The calculated front-to-back ratio affects only the midpoint of the rear lobe, pulling it inward at higher values. The rejection of most rearward QRM is most likely to depend upon the lobes and less likely to depend upon the peak front-to-back ratio figures.

Nonetheless, 20 dB of rearward rejection is admirable not only for a 2-element beam, but for any 3-element beam as well. In crowded bands, the rejection may be more important for some hams than the half dB gain advantage offered by the 2-element Yagi. These much neglected antennas very likely deserve more attention than they currently receive, even if construction requires more ingenuity. Special attention is needed on feed methods to obtain the proper phasing. Models suggest that performance does not significantly suffer as the phase angles move from about 130 degrees to nearly 140 degrees. However, achieving even this broad condition at less than the high impedances offered by folded dipole construction seems to have eluded the literature. Nevertheless, even in the abstract, these are interesting antennas to model.

Both versions of the ZL-Special reverse the patterns of gain and front-to-back ratio offered by the 2-element Yagi. Whereas the Yagi exhibits a well-behaved gain curve, the gain of the ZL-Special resembles a dipole with a rising gain figure, as Figure 21 shows. Peaks and valleys occur at the same heights as for the dipole, and in about the same amount: the 1.1 to 1.4 dB range. Gain in both models appears to be roughly inversely co-variant with the feedpoint reactance. In contrast to the 2-element front-to-back ratio curve, which shows semi-sinusoidal characteristics, the ZL-Special front-to-back ratio curves (Figure 22) are coarser, but upward bound. Model ZLSP shows some noticeable peaks and valleys which are much flattened in the FDSP curve. Nonetheless, no decline goes more than a fourth of the way down to the preceding valley, which makes the dips of little design concern. The craggy or erratic nature of the small steps in the curve make correlation with any impedance factor more speculative than certain.

The use of 135-degree phased feed systems for elements spaced 1/8 wavelength apart does not guarantee a smooth front-to-back ratio curve. As a design exercise, I made up a phase-fed double extended dou-



### 135-DEGREE PHASE-FED ANTENNAS

Antenna Description	Designator	Material	Element	Length	Space to Previous Element	Free Space Characteristics				Source
						Gain dBi	F-B dB	R	X	
"Field Day Special" (fat-wire dipole elements) 135 degree phase fed	FDSP	0.145-inch Cu	DE RefL	18.12 feet 18.12 feet	4.79 feet	5.976	22.8	29.8* 22.1*	-18.1* -99.4*	ELNEC file scaled *R&X indicators, not twinlead values
ZL-Special (fat-wire dipole elements) 135 degree phase fed	ZLSP	0.145 inch Cu	DE RefL	18.4 feet 18.9 feet	4.9 feet	6.238	42.5	26.7* 23.7*	-0.4* -50.5*	Ant Rndp, v2, p.66 *R&X indicators, not twinlead values
Double EDZ ZL 135 degree phase fed	DEDZP	#18 Cu	DE RefL	48.32 feet 51.06 feet	4.9 feet	8.898	24.8	89.3 30.9	-824 -680	Exp. design

Notes: Ant Rndp - Antenna Roundup, vol. 2, 1966, p. 66.

Table 7. 135-Degree phase-fed antennas modeled in this study.

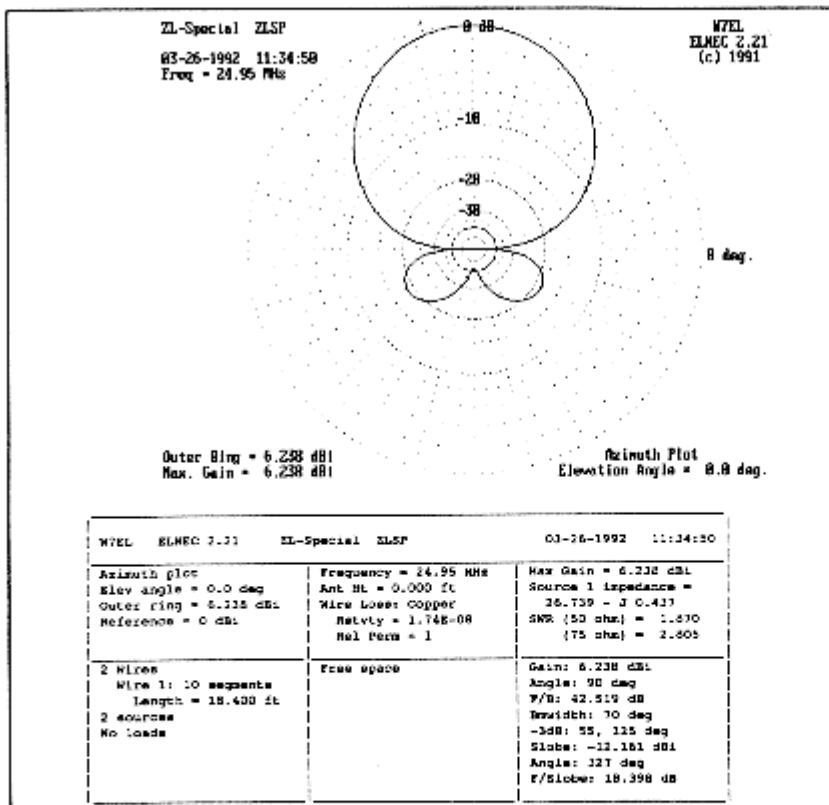


Figure 20. Free space pattern for ZLSP—a ZL-Special model using a single thick radiator to replace the folded dipole for each element.

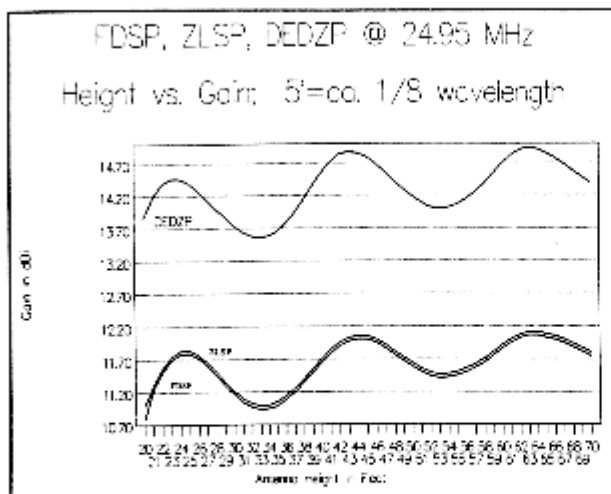


Figure 21. Gain variations in 135-degree phase-fed antennas FDSP, ZLSP, and DEDZp from 20 to 70 foot heights.

ble Zepp (DEDZP) of unequal elements. The calculated gain of the antenna appears in Figure 21 with those of the ZL-Specials. The curve parallels the lower curves in just the way in which the EDZ curve parallels those of the dipoles. The maxima and minima appear a foot or two lower, which suggests a high value of capacitive reactance at the feedpoints, verified by Table 7. Like the ZL-Specials, gain appears to be roughly in-

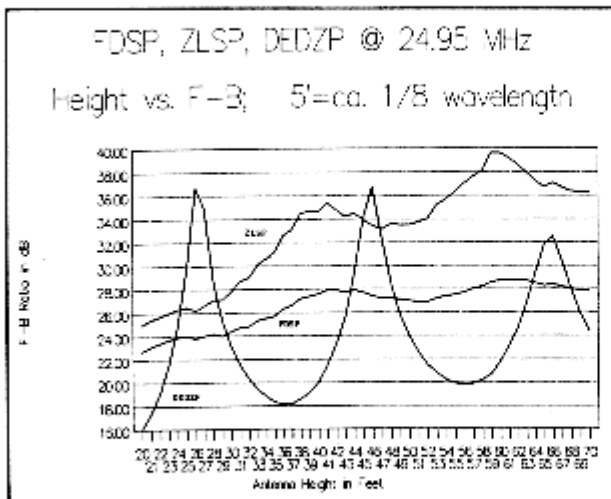


Figure 22. Front-to-back ratio variations in 135-degree phase-fed antennas FDSP, ZLSP, and DEDZp from 20 to 70 foot heights.

versely co-variant with feedpoint reactance (when we treat the values as negative numbers so that the least capacitive reactance is a maxima; that is, the most inductive reactance). Unlike its ZL-cousins, DEDZP does not exhibit a smooth front-to-back ratio curve; in fact, just the opposite. The sharp peak values in Figure 22 are exceptions, and the more average value is somewhat below the values for FDSP.

On the assumption that one can build and feed this antenna, perhaps its most significant use would be as a fixed wire beam set at a height to maximize front-to-back ratio. The high capacitive reactance of both elements strongly suggests a narrow bandwidth, and its pattern, shown in Figure 23, points to a narrow beam width. However, the multiplicity of side lobes limits absolute rejection of QRM. As a passing note, DEDZP derives from a parasitic version designed by Brian Egan, ZL1LE. It has similar gain figures, but requires an inductive load in the reflector. That factor, which requires optimizing at each height step, excluded the ZL1LE antenna from this study. However, for raw low-price gain in a fixed beam, these designs are worth considering.

#### Delta and quad loop antennas

The last group of basic ham antennas includes delta and quad loops. I have included a single delta and a single quad, each modeled as a parasitically fed and as a phase-fed beam. It is short and simple to alter the feed system in computer antenna modeling; it is the initial mutual impedance calculations that take so long. Had I used fully tapered elements to provide the most accurate dimensions and impedance figures, the task would have required over 7-1/2 hours per antenna. I cut that to about 3 hours per antenna by using substitute designs with fewer segments per wire. I already had a collection of quad and delta loop designs modeled in fully tapered form, but only for 7 steps between 25 and 55 feet. I chose the substitute designs with larger element dimensions for their relative coincidence of free space values and the accuracy of track with the tapered antennas. The delta loop model uses 8 segments per wire, while the quad model uses 6 segments per wire. The resulting patterns can be used with confidence, but the dimensions may not. Table 8 lists both the substitutes and the their more accurate models. Note that the designs were selected for their close element spacing and for resonance. A further difficulty of modeling loop beams is that most builders design them for field adjust-

ment of the reflector. That element is therefore normally too long (capacitive adjustment) or too short (inductive or fold-back adjustment) for modeling without optimizing a load for every one of the 51 height steps. The only generalization I have noted in my modeling efforts is that the traditional element formulas of  $1005/f$  and  $1030/f$  never appear in the same antenna. A comparison of QC-3 and DL-2 in Table 8 illustrates the point.

Deltas and quads present special problems for analysis. Reflected high angle radiation must intercept multiple elements at different heights. Indeed, determining the height of a delta loop and a quad is itself problematic. For convenience, I used the boom or hub altitude. The quad boom is vertically centered between elements. However, takeoff angle readings suggest that the

effective center of radiation is about a foot or 0.025 wavelength higher. Had I used this height, takeoff angles would have coincided closely with those for 2 and 3-element Yagis. Using the boom-hub height yields a takeoff angle lower than that of comparable antennas.

The situation is somewhat simpler for the delta loop. If one uses spider construction, with the triangle apex at the top (which is also the feedpoint), then the hub is about 1/3 the vertical dimension of the antenna. Using this figure, I found that takeoff angles paralleled closely with those for Yagis. Different construction methods, of course, will result in different relationships between the boom and the antenna. Because there is little difference in the patterns of a delta and a quad loop beam, Figure 23, which shows the free space pattern of the

DELTA AND QUAD LOOP ANTENNAS: (all antennas use substitute models)										
Antenna Description	Designator	Material	Element	Length	Space to Previous Element	Free Space Characteristics				Source
						Gain dBi	F-BdB	R	X	
2-element Quad (tapered element model) (parasitic values shown)	QC-3	#18 Cu	DE Refl.	39.68 feet 41.60 feet	5.0 feet	7.180	21.6	95.0	4.7	Experimental design 990/f, 1037/f substitute below
2-element Quad (6 segment/wire model) parasitic feed	QC-3A	#18 Cu	DE Refl.	(40.80 feet) (42.88 feet)	5.0 feet	7.261	22.5	96.9	4.3	Substitute model for QC-3 6 segments/wire
2-element Quad (6 segment/wire model) 135 degree phase fed	QCP-3A	#18 Cu	DE Refl.	(40.80 feet) (42.88 feet)	5.0 feet	7.203	27.3	101.8 -3.3	1.0 -4.6	Substitute model for QC-3 6 segments/wire
2-element Delta Loop (tapered element model) (parasitic values shown)	DL-2	#18 Cu	DE Refl.	40.26 feet 41.88 feet	5.0 feet	7.080	18.4	77.8	4.7	Experimental Design 1005/f, 1045/f substitute below
2-element Delta Loop (6 segment/wire model) parasitic feed	DL-2A	#18 Cu	DE Refl.	(41.40 feet) (43.26 feet)	5.0 feet	7.139	18.4	85.8	-1.7	Substitute model for DL-2 8 segments/wire
2-element Delta Loop (6 segment/wire model) 135 degree phase fed	DLP-2A	#18 Cu	DE Refl.	(41.40 feet) (43.26 feet)	5.0 feet	6.967	26.7	102.2 -2.1	-2.5 -13.1	Substitute model for DL-2 8 segments/wire

Table 8. Delta and quad loop antennas modeled in this study.

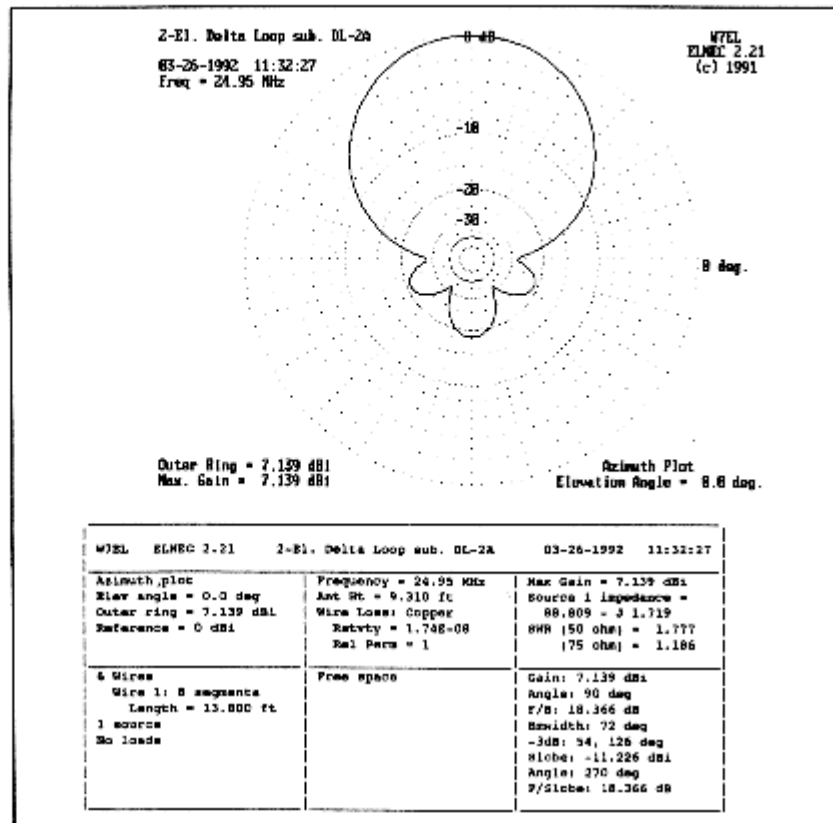


Figure 23. Free space pattern for DL-2A, a 2-element Delta Loop beam.

substitute delta loop, will suffice for both.

The gain of loop-based beams shows a more rapid and steady climb than those of Yagis, but with distinct maxima and minima. Among the antennas, there is little to choose, as the intertwining curves of Figure 24 demonstrates. The higher pair of curves belong to the quad. They are notable only for the fact that the phase-fed model manages to exceed the parasitic model in peak values at gain maxima. The differences actually make no practical difference. Maxima and minima positions coincide with those for dipoles but depart somewhat from the corresponding maxima and minima of the feedpoint resistance.

Perhaps the most important feature of the gain curves is the manner in which gain falls off below the  $5/8$  wavelength point. The reputation of the quad and the delta loop is that they are relatively immune to detuning by nearby objects and the ground. This reputation does not extend to gain. At

mounting heights below  $5/8$  wavelength, the loop beam loses its advantage over a 2-element Yagi.

With respect to front-to-back ratios, both the quad and the delta loop display very peaky patterns when parasitically fed. The two lower curves in Figure 25 show peaks similar to those in the DEDZP pattern. The quad shows an upward displacement of its peaks and valleys that appears more significant than it is actually is: the displacement would seem less had the peaks been less sharp. In the case of both antennas, however, the front-to-back ratio, even in the valleys, rivals that of a well-designed 3-element Yagi.

Some designers have suggested that quad performance might be improved by phase-feeding the rear element in the manner of a ZL-Special. The upper curves of Figure 25 show the improvement, were such a feed system to be feasible. Although the phase-fed delta loop beam displays initial "peaki-

ness," it tends to level off at the 40-foot or 1-wavelength mark. The phase-fed quad is a paradigm of a well-behaved front-to-back curve. Nevertheless, as in the case of the ZL-Specials, there may be a limit as to the usability of extreme front-to-back ratios in antennas with rear side lobes. The lobes may better mark the limits of effective QRM rejection than the tiny but deep inset of the 180-degree front-to-back point.

If MININEC programs or computers become more efficient or more automated, further study of loop beams is both desirable and necessary. A single model of each type of loop beam is insufficient to certify the patterns as reliable enough to use. Indeed, one should at least double the number of antenna models used in this study before counting its results as more than preliminary.

Nevertheless, if this study has brought about an acquaintance with and an appreciation of the ways in which antenna performance changes with antenna height, then it has been worth the time and energy. Listing antenna specifications accurately and comparing them sensibly have always been arduous and tricky tasks. Unfortunately, I have the feeling that these notes may make the tasks a bit more difficult, even if the result is a more rational set of expectations.

### Appendix 1: Take-off Angles of the Modeled Antennas

As noted early in the report, all antenna data collected refer to the lowest main lobe of radiation. As the height of any horizontal antenna is increased, the angle of maximum radiation from this lobe decreases. At the lowest heights investigated, antenna patterns show a single lobe (as viewed on an elevation plot between ground and 90 degrees straight up). As antenna height increases, other lobes appear—while the lowest lobe grows narrower. These multiple lobes may give an antenna at a set height good performance on both long skip paths and shorter domestic ones.

A full discussion of antenna elevation plots would unnecessarily lengthen this report, and several studies already exist. However, as an adjunct to this report, Table 9 reports the take-off angles of the lowest main lobe for the antennas investigated. Antennas were grouped together if they varied from the stepping points in no more than two places. To keep the table from becoming a mere morass of numbers, I have included only the angle value when it first appears as antenna height increases. The blank spaces beneath a number have the same value. The actual angle decreases

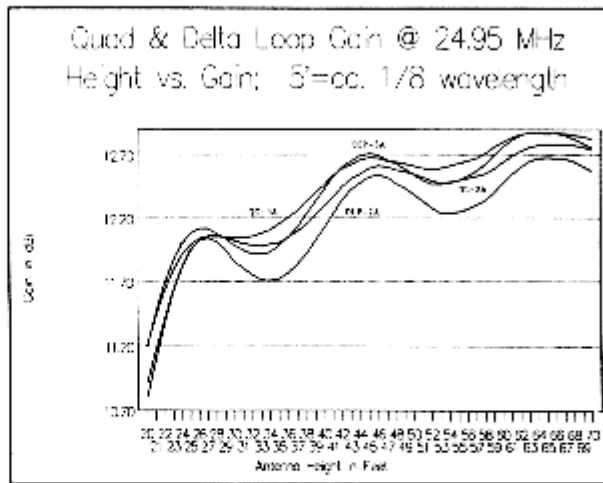


Figure 24. Gain variations in DL-2A, DLP-2A, QC-3A, and QCP-3A from 20 to 70 foot heights.

smoothly: the stepped appearance of the columns is an artifact of giving the angle value in integers.

Above three-quarters of a wavelength, the actual difference in the take-off angle among the different antennas is quite small. The dipoles, the 2-element Yagis, the 135-degree antennas, and the delta loop show a close coincidence in their values, as do the 180-degree antennas, the 3-element Yagis, and the quads. The angles shown for the delta loop and the quad are functions of their mounting points, as described above. Nonetheless, the differences between the

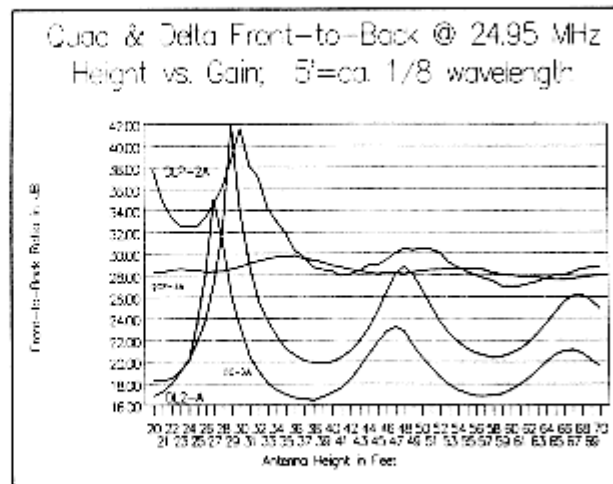


Figure 25. Front-to-back ratio variations in DL-2A, DLP-2A, QC-3A, and QCP-3A from 20 to 70 foot heights.

two groups are too small at higher mounting points to make a significant difference in antenna choice.

More significant are the differences in take-off angle for the lowest heights investigated. At 1/2 wavelength (about 20 feet at 24.95 MHz), the 4 degree difference between a dipole and a 3-element Yagi or a 2-element quad may make a significant difference in long-range performance. However, in all cases, the lobes show con-

siderable power radiation above and below the angle of maximum radiation from the lobe. In addition, evaluating a proposed antenna and mounting height requires that one consider a number of other variables, such as the desired type of operating, the overall azimuth and elevation patterns, and the ability to install and maintain the antenna. I have added these notes only to verify the validity of comparing the antennas models investigated and to give a general

TAKE-OFF ANGLES FOR THE LOWEST LOBE OF SEVERAL TYPES OF ANTENNAS										
Height		Take-off Angle in Degrees								
Feet	Wavelengths	Dipoles + EDZ	D8K + DEDZ	2-element Yagis	3-El. Yagis			FD-2LSP + DED2P	DL-2A DLP-2A	QC-3A QCP-3A
					Y3-1	Y3-2	Y3-3			
20	1/2	28	25	26	25	24	24	26	26	24
21		27	24	25	24	23	23	25	25	24
22		25	23	24	23	22	22	24	24	23
23		24	22	23	22	21	22	23	23	22
24		23	21	22	22	21	21	22	22	21
25	5/8	22	21	21	21	20	20	21	21	20
26		21	20	20	20	20				
27			19		19	19	19	20	20	19
28		20		19		18	18	19	19	
29		19	18	18	18					18
30	3/4	18				17	17	18	18	17
31		17	17	17		17	17			
32		17			16	16				16
33			16	16	16			16	16	
34		16								
35	7/8		15	15	15	15	15			15
36		15						15	15	
37			14		14	14	14			14
38				14				14	14	
39		14								
40	1		13		13	13	13			13
41				13				12	12	
42		13								
43										
44			12	12	12	12	12	12		12
45	1 1/8	12							12	
46										
47										
48			11	11	11	11	11	11		11
49		11							11	
50	1 1/4									
51										
52			10	10	10	10	10	10		10
53		10							10	
54										
55	1 3/8									
56										
57										
58			9	9	9	9	9	9		9
59		9						9	9	
60	1 1/2									
61										
62										
63										
64										
65	1 5/8		8	8	8	8	8	8		8
66		8						8	8	
67										
68										
69										
70	1 3/4									

Table 9. Comparison of antenna take-off angles for selected antennas.

impression of how take-off angle varies with antenna height.

## Appendix 2: Size, Soil, and the Limits of Extrapolation

As briefly noted in the text, the ability to extrapolate the results of this study to generalities about all HF antennas, whatever their type, is limited. The patterns are reliable at best over the range from 14 to 30 MHz, and only if one assumes an obstruction-free coherent medium or average earth beneath the antenna.

Perhaps the most obvious limitation to extrapolation is that antennas below 20 meters and above 10 meters rarely hit the elevation range used in this study. The lowest limit,  $1/2$  wavelength, is above 140 feet at 80 meters and 70 feet at 40 meters. Likewise, 20 feet is already greater than a wavelength at 6 meters. Hence, for bands outside the upper HF frequencies, the study does not cover typical antenna heights.

There are also limitations imposed by the assumption of a coherent medium or average earth beneath the antenna. Average earth is sometimes defined as having a conductivity of 5 milliSiemens per meter (mS/m) and a dielectric constant of 13. These figures represent the default values used in ELNEC 2.21, which automatically takes into account modifications to antenna patterns created by soil conditions. One may choose other soil values most like one's home QTH by modifying the ground constants in the program. Figure 26 demonstrates the changes in values for a half-wavelength dipole occasioned by selecting other earth constants. Included are perfect soil, very good soil of a rich pastoral nature ( $C = 30.3$  mS/m,  $DC = 20$ ), medium or average soil ( $C = 5$  mS/m,  $DC = 13$ ), poor sandy soil ( $C = 2$  mS/m,  $DC = 10$ ), and very poor industrial city soil ( $C = 1$  mS/m,  $DC = 5$ ).\*

The only significant change among the lines is the gain value for each given height. For a selected frequency, in this case 24.95 MHz, the maxima and minima of gain appear at the same heights above ground. As the soil grows poorer, the take-off angle of the lowest lobe of radiation decreases slightly for any elevation, but above  $5/8$  wavelength, the difference is always less than a degree. The decrease in gain more than offsets the seeming take-off angle advantage. From the comparison one may conclude that soil type, while affecting antenna gain at any height, does not affect the trends under study. Nonetheless, precise analysis for any particular site will require, for precision, replication of the exercise.

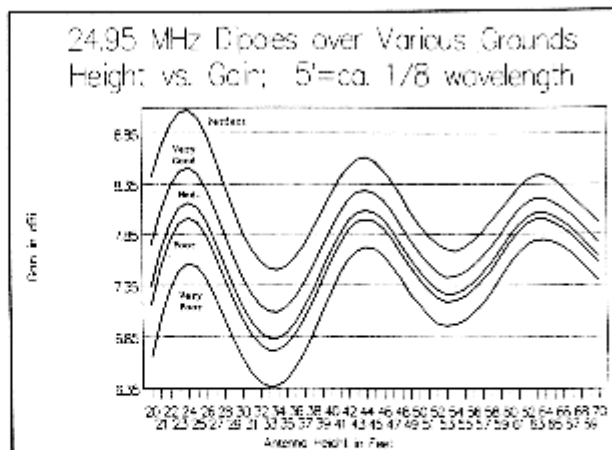


Figure 26. Gain variations in half-wavelength dipoles over various types of ground from 20 to 70 foot heights.

In addition to the differences occasioned by soil type, we must take into account the varying characteristics of soils in any typical site and the degree to which antenna currents penetrate the soils. One standard by which penetration is measured is skin depth, which increases as soils grow poorer. In the upper HF range, penetration depths to the standard measure vary from 5 to 6 feet for very good soil to nearly 40 feet for very poor soil. Consequently, scratching the surface to ordinary garden depths is insufficient to analyze the soils underlying an antenna system. Too, the assumption of coherent soil of constant characteristics is usually unwarranted by subsurface conditions.

Treating the soil as if it were a conductive surface rather than a medium is thus dangerous below the VHF range. Moreover, the degree of penetration increases rapidly at 7 MHz and below. Although the effects of ground differ according to the polarization of antennas under investigation, it cannot be ignored and becomes very significant from 160 through 40 meters. Extrapolating the results of this study to the lower HF region is thus not recommended. Rather, the study should be replicated for those frequency ranges.\*

\*These values are taken from a 1919 Federal Register or the subset of "Standards of Good Engineering Practice Concerning Standard Broadcast Stations" as reprinted by Lemay in the *Radio Engineer's Handbook* (New York: McGraw-Hill, 1953), page 709, and further reprinted in Gerard Hall, Editor, *The ARRL Antenna Book*, 16th Edition (Newington, ARRL, page 1-3).

\*Further information on antenna ground modeling can be derived from the excellent discussion set included with ELNEC as a starter. Lewallen references the current edition of *The ARRL Antenna Book*, previously noted, which in turn will lead the reader to Terman, and perhaps beyond. All of which demonstrates that the formulas required to perform this study have been around a very long time, and the difficulty of performing them manually has deterred studies of antenna performance versus height for many antenna types.